# THE USE OF VENTING FORMULAE IN THE DESIGN AND PROTECTION OF BUILDING AND INDUSTRIAL PLANT FROM DAMAGE BY GAS OR VAPOUR EXPLOSIONS 

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Summary
The use of venting formulae in explosion protection has been examined. The various types of formulae are discussed in detail and their strengths and weaknesses assessed. It is concluded that the most promising approach is a mathematical model by Yao which allows the pressure-time history of a confined or partially confined explosion to be evaluated numerically.

## Introduction

There are two types of gas explosion, namely, deflagration and detonation. These two are distinguished, principally by their flame speed and also by their structure and destructive force. A deflagration involves a flame travelling through a combustible mixture at less than the ambient speed of sound in the mixture, whilst in a detonation the flame front is coalesced with a shock wave and travels at speeds whose values can be several times greater than the speed of sound in the medium at normal temperatures and pressures. Since pressure is transmitted at the local speed of sound of the medium, it is impossible to protect plant from the effects of detonations by the means of venting. This report will, therefore, be restricted to a consideration of deflagration explosions only. Fortunately detonations in gases or vapours are comparatively rare and are only likely to be experienced when the source of the ignition generates strong shock waves; or the explosion occurs in a long pipe or duct, so that the flame has an opportunity of accelerating along the pipe and generating a strong shock wave; or when highly turbulent conditions prevail and/or the volumes of combustible gas involved are substantially larger than the volumes considered in existing venting formulae; or when mixtures of high burning velocity are present.

The use and validity of venting formulae will receive particular attention, especially where the predictions from such formulae are relevant to confined explosions in relatively large volumes ( $>600 \mathrm{~m}^{3}$ ). To facilitate the use of this
report, formulae will be classified according to their type and origin rather than on any strictly chronological basis.

## Venting

The most common method of protecting structures from the effects of internal explosions is by means of pressure relief, by venting. Other means of protection of installations do of course exist, such as inerting with noncombustible gases (typically nitrogen or carbon dioxide) or using suppression systems. The latter depend on the prevention of explosion development by the use of either chemical inhibitors, especially halocarbons, or in some cases by means of materials which physically suppress the explosion reaction, e.g. water sprays or dry powders such as stone dust in coal dust explosions (all of the latter methods are commonly used for control in mines). These methods however, are generally less widely applicable. Thus they tend to be used either in conjunction with venting or by themselves when venting is not possible, as may be the case when an installation is below ground or the materials being handled are too toxic to permit them to be vented.

Venting is possible because fundamental burning velocities (i.e. the maximum velocity with which a plane flame front moves normal to its surface through the adjacent quiescent unburnt gas) are normally of the order 1-10 $\mathrm{m} / \mathrm{s}$ (most hydrocarbon/air mixtures have fundamental burning velocities less than $1 \mathrm{~m} / \mathrm{s}$ although flame speeds may be considerably higher [1]) while the velocity of sound in air and most gaseous media is around $340 \mathrm{~m} / \mathrm{s}$ at normal temperatures and pressures. This means that, except for all but the largest structures, pressure transmission may be regarded as effectively instantaneous but the rate of pressure rise will be relatively slow, subjecting the structure to be protected uniformly to a stress fixed by the amount of combustible material. Vents may therefore be located with equal value wherever feasible in a compartment or duct, providing the run-up distances from the point of ignition to the vents are similar.

The maximum value of the explosion pressure in a confined deflagration explosion will depend on the relieving pressure of the vent, the time it takes to operate, the burnt gas temperature (any consequent increase in energy causes increases in pressure when confined and expansion when unconfined), the initial pressure and temperature of the gas, the amount of flammable gas and its composition and the rate of loss of heat to the walls of the vessel. The latter is relatively slow and an explosion may be regarded as an effectively adiabatic process.

Completely confined explosions in compartments of dimension likely to be encountered in domestic and industrial premises are reasonably well understood [2] and will not be considered further, in that most plant and domestic structures are unlikely to withstand them. When an explosion is vented the important parameters for a given risk are the size of the vent relative to the vessel or duct, the scale and intensity of turbulence and the nature of the
ignition source (e.g. multiple ignition sources often produce higher explosion pressures than single ignition sources [3]). The position of the ignition source relative to the vent is also important. However, since it is not possible to guarantee the position of an accidental ignition source, central ignition, which is likely to produce the highest pressures in spherical or rectangular structures, is normally assumed in most treatments so as to cater for the worst possible situation.

For a non-vented gas explosion, with ignition at the centre of a spherical enclosure, the pressure-time history will be of the form indicated by curve ' $A$ ' in Fig. 1. The pressure will increase, at first slowly and then more and more rapidly as the flame front comes into contact with an increasingly large surface area of unburnt gas. Finally the pressure will reach a maximum value as the flame front touches the walls and then slowly diminish as heat is dissipated through the walls of the vessel.

In a moderately large vessel $\left(10^{2}-10^{3} \mathrm{~m}^{3}\right)$ the overall process may take several seconds to complete. The maximum explosion pressure $P_{\text {max }}$ may have a value of around $700 \mathrm{kN} / \mathrm{m}^{2}$ and at this pressure all but certain specially designed constructions, such as a pressure vessel, are likely to be demolished.

The effects of explosion pressures on personnel is discussed elsewhere [4]. However in general humans can survive pressures greater than those that damage structures.

When a pressure relief is installed in a vessel, operating at a pressure $P_{\mathrm{v}}$, the pressure in the vessel may still continue to rise for $100 \mathrm{~m} / \mathrm{s}$ or more to a peak value $P_{1}$ when the effect of the vent causes $\mathrm{d} p / \mathrm{d} t$ to become negative. There-


Fig. 1. A representation of a pressure-time history of an unvented (curve A) and vented (curve B) deflagrative explosion.
after one might expect the pressure to fall to ambient pressure or below, but since the flame front is still expanding while the explosion proceeds $\mathrm{d} p / \mathrm{d} t$ may become positive again and the pressure will rise. If this effect predominates over the effect of the rate of loss of material through the vent then $\mathrm{d} p / \mathrm{d} t$ will continue to increase until the flame front reaches the surface of the vessel and a second pressure peak $P_{2}$ will occur. The pressure-time history will thus take the form of Curve B in Fig. 1.

A further pressure peak $P_{3}$ may occur if the gas cools down sufficiently rapidly to reduce the pressure in the vessel below ambient, causing re-entry of ejected unburnt gases. Transient pressure may also be affected by oscillations (spiking) being set up in the system [5]. However, neither these nor the third pressure peak $P_{3}$ are likely to be important in terms of their ability to damage the structure in which they occur providing the vents cannot reseal (e.g. hinged or spring loaded vent covers).

The relative sizes of peaks 1 and 2 are determined by the size of the vent relative to the vessel, the magnitude of $P_{v}$, the flame speed and the scale and intensity of turbulence set up when the vent operates. An increase in $P_{\mathrm{v}}$ will cause $P_{1}$ to increase with respect to $P_{2}$; while high flame speed and high turbulence such as that generated by the operation of bursting disc vents [6], will cause $P_{2}$ to increase with respect to $P_{1} . P_{2}$ will also increase with respect to $P_{1}$ as the vent size becomes smaller for a given vessel.

The separate peaks associated with $P_{1}$ and $P_{2}$ may, depending on a variety of factors, merge completely. The merging of $P_{2}$ with $P_{1}$, and of $P_{1}$ with $P_{2}$ may be distinguished physically. The merging of $P_{2}$ with $P_{1}$ corresponds to a situation where $\mathrm{d} p / \mathrm{d} t$ becomes negative after the operation of the vent. This will occur when the vent is relatively large or the flame speed low. The merging of $P_{1}$ with $P_{2}$ corresponds to the situation where $\mathrm{d} p / \mathrm{d} t$ remains positive after the opening of the vent and will occur when the vent is relatively small and flame speeds are high.

The designer of industrial plant or domestic structures will be interested in the maximum pressure $P_{\max }$ whether it be due to $P_{1}$ or $P_{2}$. Ideally the designer requires the value of $\int_{t_{1}^{2}}^{t_{2}} P_{\max } \mathrm{d} t$ sometimes called the pressure impulse (where $t_{1}$ and $t_{2}$ are the times at which the structure is subjected to pressure greater than the maximum static pressure its weakest member can withstand, so as to design structures in which the weakest part can withstand the pressure impulse). However, there is little information on this aspect of the explosion resistance of buildings and plant and in most attempts to produce venting formulae the difference between static and dynamic pressure is ignored. Because buildings and plant can normally withstand higher transient than static pressures, designing for static pressure incorporates a safety factor.

There have been numerous attempts to produce venting formulae which can predict the values of $P_{1}$ or $P_{2}$ and their dependence on the various explosion parameters, especially venting parameters. These formulae and their associated difficulties are discussed below, special attention being paid to those approaches likely to predict values of $P_{\max }$ in very large volumes
(i.e. $600 \mathrm{~m}^{3}$ or above). Practical experiments are too difficult and expensive for such large volumes to justify many tests, and a proven theoretical or empirical method of providing solutions is eminently desirable.

## Venting formulae

Some of the earliest work was carried out by Cubbage and Simmonds $[7,8]$ who measured the explosion pressures generated in large box-ovens (of volumes up to $14 \mathrm{~m}^{3}$ ) by town gas/air mixtures using central ignition.

They attempted to relate the magnitude of the maximum pressure associated with each pressure peak, i.e. $P_{1}$ and $P_{2}$, to:
(a) the volume of the vessel $V$;
(b) the weight per unit area of the vent $W$; and
(c) the size of the vent, expressed in terms of a venting ratio $K$, defined as
the the ratio of cross-sectional area $A_{c}$ of the side of the oven where the vent was installed and the area of the vent $A_{\mathrm{v}}$ i.e. $K=A_{\mathrm{c}} / A_{\mathrm{v}}$.
They were able to show the following relationship held for their experiments:
$P_{1}=(g K W+m) V^{-\frac{1}{3}}$
where $g$ and $m$ are constants for any particular flammable gas/air mixture while for town gas the second pressure peak $P_{2}$ was numerically equal to seven times the venting ratio, $K$, when $P_{2}$ was expressed in $\mathrm{kN} / \mathrm{m}^{2}$ i.e.
$P_{2}=7 K$
It is now customary to define $K$ as the ratio of the area of the side of minimum cross-section to the area of the vent. As Cubbage and Simmonds invariably placed their vents in the side of minimum cross-section the alternative definitions of $K$ are equivalent.

Both formulae were found to hold over a wide range of conditions. Eqn. 1 was shown to be valid for values of $V$ of 0.23 to $4.08 \mathrm{~m}^{3}$, values of $K$ from 1 to 4 and of $W$ from 8.8 to $34.2 \mathrm{~kg} / \mathrm{m}^{2}$, but eqn. 2 held over a smaller range of $K$ as $P_{2}$ began to show some dependence on the oven volume when $K$ was greater than 2.5. It was also found that the pressure peak $P_{1}$ showed direct dependence on the burning velocity $S$ of the gas/air mixture employed. A modified form of eqn. 1 was therefore proposed:
$P_{1}=S\left(g_{1} K W+m_{1}\right) V^{-\frac{1}{3}}$
where $g_{1}$ and $m_{1}$ are constants for any particular gas/air mixture. For a 25 per cent town gas/air mixture which has a burning velocity of $1.2 \mathrm{~m} / \mathrm{s}$, the two constants $g$ and $m$ for eqn. 1 have values 0.51 and 3.34 and $g_{1}$ and $m_{1}$ for eqn. 3 have values of 0.43 and 2.78 respectively providing that $P_{1}$ is expressed in $\mathrm{kN} / \mathrm{m}^{2}, V$ in $\mathrm{m}^{3}$ and $W$ in $\mathrm{kg} / \mathrm{m}^{2}$.

Cubbage and Simmonds extended their work [8] up to volumes $14 \mathrm{~m}^{3}$ and found that these equations still held, even with the lower limit for $W$. In
these latter experiments $K$ had values in the range of 1 to 3 and $W$ in the range of 1.5 to $34 \mathrm{~kg} / \mathrm{m}^{2}$.

Some doubt may be cast on the way the dependence of $P_{1}$ on $S$ has been introduced in eqn. 3 since there is no a priori reason to suppose that $g$ and $m$ will have the same value for other combustible mixtures or that they will show an identical dependence on $S$.

Cubbage and Simmonds' work has also been employed in a paper produced under the auspices of the British Ceramic Research Association [9]. Once again reference is made to eqn. 3 for explosions generated in a 25 per cent town gas/air mixture, which is given as:
$P_{1} V^{\frac{1}{3}}=S(0.43 K W+2.78)$
while eqn. 2 is also presented as:
$P_{2}=5.8 \mathrm{~S} \mathrm{~K}$
$P_{1}$ and $P_{2}$ being expressed in $\mathrm{kN} / \mathrm{m}^{2}, S$ in $\mathrm{m} / \mathrm{s}, W$ in $\mathrm{kg} / \mathrm{m}^{2}$ and $V$ in $\mathrm{m}^{3}$. The use of these equations in this form still remains open to the criticism raised above.

If reference is made to the original work of Cubbage and Simmonds [7] the extent of the proportionality between the peak pressure and the burning velocity of the particular gas/air mixture employed may be questioned. Thus if we examine Table 1 (for which the data are taken from ref.7) we see that values of $P_{2}$ predicted by eqn. 5 are 27 per cent lower than the measured value with an actylene/air mixture and 25 per cent above the measured value for $P_{2}$ with a carbon disulphide/air mixture.

Cubbage and Simmonds have also reported [7] that they obtained anomalous results when using eqn. 4 to predict values of $P_{1}$ for an acetylene/ air mixture, the actual measured value of $P_{1}$ being 60 per cent greater than the predicted one. This seems to indicate that some caution must be used in the application of eqns. 4 and 5 to mixtures containing different flammable gases.

Cubbage and Simmonds reported that the second pressure peak was frequently greater than the first [7]. It may be shown from eqns. 4 and 5 that $P_{2}$ is less than $P_{1}$ when $V$ is greater than $(0.074 W+0.3834)^{3}$ regardless of the size of the vent ( $V$ in $\mathrm{m}^{3}$ and $W$ in $\mathrm{kg} \mathrm{m}^{-2}$ ).

TABLE 1

| Per <br> cent | Gas mixture | $K$ | $P_{2}$ (measured) <br> $\left(\mathrm{kN} / \mathrm{m}^{2}\right)$ | $S$ <br> $(\mathrm{~m} / \mathrm{s})$ | $P_{2}$ (predicted) <br> $\left(\mathrm{kN} / \mathrm{m}^{2}\right)$ |
| :---: | :--- | :--- | :---: | :--- | :--- |
| 25 | Town gas/air | 1.25 | 9.08 | 1.19 | 8.73 |
| 10 | Methane/air | 1.25 | 2.79 | 0.57 | 2.65 |
| 8 | Carbon disulphide/air | 1.25 | 2.79 | 0.49 | 3.49 |
| 9.4 | Acetylene/air | 1.25 | 11.89 | 1.31 | 9.36 |

Thus for the vents made of the heaviest material used ( $W=34.2 \mathrm{~kg} / \mathrm{m}^{2}$ ) $P_{2}$ is less than $P_{1}$ unless V is greater than $25 \mathrm{~m}^{3}$ while for a light vent material ( $W=1.46 \mathrm{~kg} / \mathrm{m}^{2}$ ) and the maximum vent ratio $K=1, P_{2}$ is greater than $P_{1}$ unless $V$ is less than $0.2 \mathrm{~m}^{3}$. This is as one would expect when using a flangedlid type vent, since the lighter the vent the earlier it operates (i.e. $W$ is related to the operating pressure of the vent). However, it is by no means automatic that $P_{2}$ is greater than $P_{1}$, although for the lighter venting materials and higher values of $K$ this is probable.

The British Ceramic Research Association also attempted to produce a relationship predicting the maximum pressure $P_{2}^{1}$ produced in a gas filled room connected and adjacent to another, similarly gas filled, in which ignition originally occurred. They produced a formula of the form:
$P_{2}^{1}=\alpha P_{1}+\beta P_{1}^{2}$
where $P_{1}$ is the pressure produced in the first room where ignition occurred and $\alpha$ and $\beta$ are constants. Under the conditions of their work and with $P_{1}$ and $P_{2}^{1}$ expressed in $\mathrm{kN} / \mathrm{m}^{2}, \alpha$ and $\beta$ had values of 0.59 and 0.163 respectively. Unfortunately this relationship was based on only four results, one of which ( $P_{1}=21 \mathrm{kN} / \mathrm{m}^{2}, P_{2}^{1}=83.83 \mathrm{kN} / \mathrm{m}^{2}$ ) was estimated from the consequences of the Ronan Point explosion. If true this equation is of some importance since it indicates that explosion pressures experienced in a series of adjacent and connected gas-filled rooms will increase in an almost exponential manner, i.e. a cascade effect. A simple quadratic treatment of eqn. 6 together with regression analysis indicates that the limited range of data can be fitted very well (the correlation coefficient having a value of 0.98 ) to the form:
$Y=a_{0}+a_{1} X$
where $Y=\left(1 / \beta \times P_{2}+\alpha^{2} / 4 \beta^{2}\right)^{\frac{1}{2}}$ and $X=\left(P_{1}+\alpha / 2 \beta\right), a_{0}=-0.1$ and $a_{1}=1.04$ (instead of the ideal values of 0 and 1 respectively). This agreement is probably as good as one might hope to obtain; recently however, Cubbage and Marshall [10] have queried this equation because it gives a wrong dependence on the burning velocity. Instead they have suggested a new equation, namely:
$P_{2}=\left(a P_{1}+b P_{1}^{2}\right)^{\frac{1}{2}}$
where $a$ and $b$ contain terms dependent on the structure and the gas/air mixture involved, given by:

$$
\begin{align*}
a & =\left(\frac{V_{2}}{V_{1}}\right) \frac{(K W)_{2, \mathrm{av}} S^{2}}{V^{\frac{1}{3}}}  \tag{9}\\
b & =\left(\frac{V_{2}}{V_{1}}\right) \frac{K_{2}}{K_{1,2}} \tag{10}
\end{align*}
$$

where $V_{1} \quad=$ the volume of room 1
$V_{2} \quad=$ the volume of room 2
$(K W)_{2, \mathrm{av}}=$ the average value of the term (KW) for the second room, which is the room into which the explosion propagates (see later for complete explanation, i.e. eqn. 13)
$K_{1,2}=$ the venting ratio between rooms 1 and 2
$K_{2} \quad=$ the venting ratio for room 2
$S \quad=$ the burning velocity of the gas/air mixture employed
Since this equation does not predict the same dramatic increase in pressure given by eqn. 6 via the 'cascade' effect it is important to determine the extent to which such an effect can occur, for which further work is necessary.
In a later publication [11] further work is reported in which explosions occurring in one of a pair of adjoining rooms gave rise to three pressure peaks in the second room. The first peak resulted from the pressure rise in the first room, the second from the subsequent explosion in the second room and the third peak, which showed considerable spiking*, probably arose from the oscillations of the gas between the rooms. The origin of the third peak is not clear but as it appears to be much smaller than the second pressure peak, it is not of significance in practical venting problems.

The work of Cubbage and Marshall [10] also contains a modified venting formula based on earlier work of Cubbage and Simmonds [7,8] and that of Rasbash et al. [12,13]. Support for their formula was supplied by data taken from a variety of enclosures ranging from a $0.9 \mathrm{~m}^{3}$ cubical steel box to a full scale load-bearing building with 4 rooms each of $28 \mathrm{~m}^{3}$. By using the method of dimensions and assuming that $P_{\mathrm{v}}=\mathrm{f}(K, W, S, V)$ they conclude that:
$P_{\text {max }}=P_{\mathrm{v}}+B K W S^{2} / V^{\frac{1}{3}}$
where $B$ is a constant.
Such an approach deserves comment in that $K$ is dimensionless and it is therefore impossible to learn anything about the nature of the dependence of $K$ on ( $P_{\max }-P_{\mathrm{v}}$ ) by such an analysis. Also the analysis is only likely to be valid if all important parameters have been included; one significant parameter that has not been included for instance is a term to account for the effect of turbulence.

From an examination of their own data they conclude that when $P_{\max }$ and $P_{\mathrm{v}}$ are expressed in $\mathrm{kN} / \mathrm{m}^{2}, W$ in $\mathrm{kg} / \mathrm{m}^{2}, S$ in $\mathrm{m} / \mathrm{s}$ and $V$ in $\mathrm{m}^{3}$, the constant $B$ has a value of 2.35 . Thus eqn. 11 becomes:
$P_{\text {max }}=P_{\mathrm{v}}+\frac{2.35 \mathrm{KW} \mathrm{S}}{}{ }^{2}$
In order to take into account the situation where there is more than one

[^0]vent, they also defined a new parameter (KW) by analogy with the case of parallel resistors, such that:
$\frac{1}{(K W)_{\mathrm{av}}}=\sum_{j=1}^{n} 1 /(K W)_{j}$
It was concluded that such an approach would be valid providing all the vents operated at similar pressures. Finally another parameter $\mathrm{F}\left(E, E_{0}\right)$ was introduced into the equation to deal with the situation when the structure is not completely full of the explosive gas mixture but is present as a pocket or layer, such as might occur with an accidental release of gas. Thus eqn. 12 becomes:
$P_{\text {max }}=P_{\mathrm{v}}+\left(2.35 \mathrm{KW} \mathrm{S} \mathrm{S}^{2} / V^{\frac{1}{3}}\right) \mathrm{F}\left(E, E_{0}\right)$
where $E$ is the actual energy in the mixture and $E_{0}$ the amount required to remove the vent, so that $\mathrm{F}\left(E, E_{0}\right)$ is a measure of the energy contained in the gas/air mixture in excess of that required to remove the vent.

Cubbage and Marshall were unable to define $\mathrm{F}\left(E, E_{0}\right)$ precisely but concluded that an appropriate form of the equation when the vent had a moderate or low operating pressure ( $P_{\mathrm{v}}$ less than $35 \mathrm{kN} / \mathrm{m}^{2}$ ) would be:
$\mathrm{F}\left(E, E_{0}\right)=1-\exp -\left(\frac{E-E_{0}}{E+E_{0}}\right)$
For vents with a high breaking pressure ( $P_{\mathrm{v}}$ greater than $35 \mathrm{kN} / \mathrm{m}^{2}$ ) eqn. 15 consistently underestimates the pressure achieved (typically by 15 per cent) and Cubbage and Marshall recommended instead that $\mathrm{F}\left(E, E_{0}\right)$ be represented as:
$\mathrm{F}\left(E, E_{0}\right)=\frac{E-E_{0}}{E}$
Ideally $\mathrm{F}\left(E, E_{0}\right)$ should adopt values of unity and zero when $E$ is much greater than $E_{0}$ and $E=E_{0}$ respectively. Although eqn. 16 meets this demand, eqn. 15 does not since it tends to ( $1-1 / e$ ) or approximately 0.63 when $E$ is much greater than $E_{0}$. Thus it will tend to underestimate pressures for higher concentrations of gas by a factor of 50 per cent and it was therefore suggested that in situations where $E$ was expected to be much greater than $E_{0}$, the original form of the equation i.e. eqn. 12 be used i.e. when the structure was likely to be completely or substantially full of flammable gas/air mixture.

When the ratio $\left(E-E_{0}\right) /\left(E+E_{0}\right)$ is much less than 1 which will occur when $E$ tends to $E_{0}$,
$\mathrm{F}\left(E, E_{0}\right) \approx \frac{E-E_{0}}{E+E_{0}} \approx \frac{E-E_{0}}{2 E} \approx \frac{E-E_{0}}{2 E_{0}}$

Now this may be expected to occur either when there is very little flammable gas in the structure or with vents having high operating pressures. Given that eqn. 16 represents the best form of $\mathrm{F}\left(E, E_{0}\right)$ for vents of high operating pressures then eqn. 15 would be expected to underestimate pressures produced, as $P_{\mathrm{v}}$ tends to $35 \mathrm{kN} / \mathrm{m}^{2}$ (and hence $E$ tends to $E_{0}$ ) and could be in error by as much as 50 per cent. Some caution, therefore, needs to be exercised in the use of these alternative expressions for $\mathrm{F}\left(E, E_{0}\right)$.

Cubbage and Marshall have also specified the conditions over which they expect their equation to be valid. These conditions are (a) the ratio of the maximum and minimum dimensions of the confining structure, i.e. the ratio of the areas of the maximum and minimum sides in a rectangular structure is less than 3 , (b) $P_{\mathrm{v}}$ is less than $49 \mathrm{kN} / \mathrm{m}^{2}$, (c) $K$ has values from 1 to 10 , (d) $W$ has values from 2.4 to $24 \mathrm{~kg} / \mathrm{m}^{2}$ and (e) $(K W)$ is less than $73 \mathrm{~kg} / \mathrm{m}^{2}$. They conclude that these conditions are likely to cover most structures met in practice.

Another attempt to relate the maximum pressure $P_{\max }$ produced in an explosion is due to Rasbash et al. [12,13] who developed an empirical formula of the type:
$P_{\max }=A P_{\mathrm{v}}+B K$
where $P_{\mathrm{v}}$ is the operating pressure of the vent, $A$ and $B$ are constants (whose values depend on the nature of the gas mixture and other parameters such as turbulence) and $K$ is the venting ratio.

This equation has some similarity with eqn.1. Perhaps the most significant difference is the absence of a term for the volume of the structure. In considering eqn. 18 a number of points need to be mentioned. The first is that this equation is entirely empirical and although it is reasonable to assume that $P_{\max }$ varies with $K^{n}$ (where $n$ is some positive number), Fig. 1 shows that the dependence of $P_{\text {max }}$ on $P_{\mathrm{v}}$ is not nearly so obvious. Thus whereas $P_{\mathrm{v}}$ will set the lower limit of the value of first pressure peak $P_{1}$, the lower its value the greater the likely magnitude of any second pressure peak. Thus while eqn. 18 may well predict the explosion pressures for a specific range of conditions, with an appropriate choice of $A$ and $B$ its predictions for other conditions cannot automatically be relied on.

Rasbash obtained values for $A, B$ for a stoichiometric propane-air mixture for values of $W$ up to $24 \mathrm{~kg} / \mathrm{m}^{2}, P_{\mathrm{v}}$ up to $7 \mathrm{kN} / \mathrm{m}^{2}$ and $K$ from 1 to 5 . The values of $A$ and $B$ under these conditions were 1.5 and 3.5 respectively. It was noted however that explosion pressures were often much greater than those predicted by eqn. 18 in conditions where a high turbulence intensity was likely, e.g. in a partitioned compartment Rasbash found $P_{\max }$ was up to 2.5 times greater. Rasbash in fact argues that the term $B$ is related to the burning velocity $S$ and hence to the scale and intensity of turbulence. Unfortunately however, no procedure seems to have been devised to provide an appropriate correction factor for the dependence of $B$ on the scale and intensity of turbulence likely to be produced in any particular structure, nor
in fact does any attempt seem to have been made to demonstrate the dependence of $B$ on $S$.

Given the direct dependence of $B$ on $S$, eqn. 18 may be written as:
$P_{\text {max }}=A P_{\mathrm{v}}+B^{\prime} S K$
and providing that the constants $A$ and $B^{\prime}$ do not change significantly from system to system we may write eqn. 19 as:
$P_{\text {max }}=1.5 P_{\mathrm{v}}+7.7 \mathrm{SK}$
$P_{\text {max }}$ and $P_{\mathrm{v}}$ being expressed in $\mathrm{kN} / \mathrm{m}^{2}$ and $S$ in $\mathrm{m} / \mathrm{sec}$.
Rasbash appears to have used this approach in producing a modified form of his equation for natural gas/air explosions ( $S_{0}=0.36 \mathrm{~m} / \mathrm{sec}$ ) of the form
$P_{\text {max }}=1.5 P_{\mathrm{v}}+2.8 \mathrm{~K}\left(\mathrm{kN} / \mathrm{m}^{2}\right)$
The work of Rasbash has recently been re-evaluated by Butlin and Tonkin [14]. These workers have carried out experiments on the explosion of natural gas/air layers in a rectangular chamber of total volume $28 \mathrm{~m}^{3}$. Although they have not worked with the chamber full of a stoichiometric gas/air mixture it has been possible to obtain the appropriate data by extrapolation from their results. They conclude that their results do not correspond to those expected from eqn.21. Values of $P_{\max }$ derived from this work and values obtained

TABLE 2
Comparison of values of $P_{\text {max }}$

from other empirical equations discussed here are given in Table 2 for comparison.

As Table 2 indicates, pressures derived from Rasbash's formula do not significantly differ from the extrapolated values of pressure for this limited set of results. However, the use of laminar burning velocities to describe explosions where significant turbulence may occur is questionable. It may well be best to write eqn. 18 as:
$P_{\text {max }}=A P_{\mathrm{v}}+n B^{\prime \prime} S K$
where $B^{\prime \prime}$ is a constant and $n$ is a turbulence factor having a value of 1 for quiescent mixtures and low turbulence conditions. Since some difficulty is likely to be experienced in choosing values for $n$, it may be considered simpler to use a more rigorous approach with theoretical justifications and therefore of positive predictive value.

Another early attempt to produce a venting formulae was due to Burgoyne and Wilson [15]. They carried out work using various n-pentane/air mixtures in two cylinderical vessels, with diameters of 1.3 m and length to diameter ratios of $1: 1$ and $3: 1$ and total capacity of 1.7 and $5.7 \mathrm{~m}^{3}$ respectively, under fairly turbulent conditions. It was found that the pressures produced could be related by an equation of the type:
$P_{\text {max }}=M \log A / a-N$
where $M$ and $N$ are constants having values of 475 and 200 when the pressure is expressed in $\mathrm{kN} / \mathrm{m}^{2}$, ' $A$ ' is the cross-sectional area of the vessel and ' $a$ ' is the relief area. Thus eqn. 23 may be written as:
$P_{\text {max }}=M \log K-N$
This type of function has an attractive feature that is not present in eqns. 11 and 18. Thus eqn. 24 predicts that $P_{\max }$ is going to be relatively insensitive to changes of $K$ for large values of $K$, as $\mathrm{d} P_{\text {max }} / \mathrm{d} K$ equals $M / K$, whereas eqns. 11 and 18 predict that $\mathrm{d} P_{\max } / \mathrm{d} K$ equals some constant. However, such an equation is clearly unable to deal with situations where venting is good and $K$ adopts low values. Since $P_{\text {max }}$ cannot adopt negative values the equation will break down if $K$ is less or equal to $10^{N / M}$ (for Burgoyne's experimental system this would be at $K=2.6$ ).

By setting $M=27$ and $N=0$ this formula provides a reasonable fit to the data of Butlin and Tonkin [14] as Table 2 indicates. This fact outlines the difficulty of such formulae. For providing one has a function which gives a smooth curve, then by appropriate choice of constants it is nearly always possible to obtain a reasonable fit for a limited range of conditions and data. Later workers do not appear to have produced equations with a logarithmic dependence of $K$ on $P_{\max }$.

A more recent formula has been produced by Dragosavic [5] for a cubical brick structure of approximate volume of $66 \mathrm{~m}^{3}$. This structure was divided into two compartments of dimensions 4 m wide, 3.5 m long and 4 m wide,

2 m long (the height of the compartments is not given but can be estimated as approximately 3 m from the photographs supplied in the report). Thirtyfour explosions were carried out, mostly with a stoichiometric natural gas/air mixture. The maximum value of the first pressure peak $P_{1}$ was estimated as:
$0.8 P_{1}=P_{1}^{\prime}=3+P_{\mathrm{v}}\left(\mathrm{kN} / \mathrm{m}^{2}\right)$
and the second as:
$0.8 P_{2}=P_{2}^{\prime}=3+0.5 P_{\mathrm{v}}+\frac{0.04}{\psi^{2}}\left(\mathrm{kN} / \mathrm{m}^{2}\right)$
where $\psi$ is a volume venting coefficient equal to the area of the vent over the volume of the enclosure. The factor of 0.8 has been introduced to convert the dynamic pressure $P_{1}$ and $P_{2}$ to a static pressure equivalent, $P_{1}^{\prime}$ and $P_{2}^{\prime}$.

These equations can be generalised as:
$P_{1}^{\prime}=A+P_{\mathrm{v}}$
$P_{2}^{\prime}=A+B P_{\mathrm{v}}+C^{\prime} K^{2}$
where $A$ and $B$ are constants and $C^{\prime}$ is a constant dependent on the composition and proportion of the gas mixture employed and the square of the greatest linear dimension of the structure concerned.

These equations realistically predict a positive pressure even in the situation where the vent is open and in this form also correctly indicate that explosion pressure ought to be dependent on the length or depth of the structure. Hence eqns. 26 and 28 are likely to overestimate the pressure produced for low values of $\psi$ and high values of $K$. Thus Table 2 (which shows the predicted values of $P_{\text {max }}$ obtained with Dragosavic formula and Butlin and Tonkin's data [14]) indicates, as might be expected, an increasingly poor agreement between the predicted and measured pressures as $K$ increases.

Perhaps the best way of assessing the validity of the various formula can be seen from Fig. 2.

The results presented in Fig. 2 are taken from the work of Cotton and Cousins [16] for a 5 per cent propane/air explosion in a tank of volume $1 \mathrm{~m}^{3}$ containing an open vent. Curves from the various formulae have been included in Fig. 2 using the constants given. The logarithmic plot of the type employed by Burgoyne and Wilson is of the same form as the experimental curve and is able to follow the experimental curve for the range of values of $K$ used. A better fit is provided by an equation suggested by Trense [35].

One of the difficulties of using these formulae arises from the number of factors likely to affect the dependence of $P_{\text {max }}$ on $K$. Thus it seems reasonable that curves showing the dependence of $P_{\max }$ on $K$ is low and the most important factor in determining the magnitude of $P_{\max }$ is the inertia of the gas; however, $P_{\max }$ will develop rapidly with increases of $K$. The second region is one in which $P_{\max }$ is principally influenced by the value of $K$ and over which the work of Rasbash et al. indicates that curves of the type $P_{\max }=$


Fig. 2. A comparison between the predicted maximum explosion pressures for various vent sizes given by empirical formulae and experimental data.
$A+B K$ can be satisfactorily employed. While the third region is one in which $P_{\text {max }}$ is tending to a limit, i.e. that of a closed container, which is determined by the chemical energy and mass of reactant, and is therefore relatively insensitive to changes in $K$.

The presence of a vent does not basically alter this analysis but rather tends to shift the values of $P_{\max }$ into the third region. Thus for example, a massive vent cover, or one with a high operating pressure, is liable to allow $P_{\max }$ to approach its limit before the vent opens so that the dependence of $P_{\max }$ on $K$ is that typified by the third region. If this analysis is correct then no one
single simple function of $K$ is likely to predict the value of $P_{\max }$ on $\dot{K}$ over all three regions.

Another approach has been developed by Decker [17] and depends on the evaluation of two equations, which have been produced assuming central ignition. These are:
$Q=\frac{R V}{P}=\frac{V}{t}$
$A=\frac{R V(M / T)^{\frac{1}{2}}}{30 P}=\frac{V(M / T)^{\frac{1}{2}}}{30 t}$
where $Q=$ volumetric flow rate at the relieving conditions ( $\mathrm{m}^{3} / \mathrm{s}$ )
$R=$ maximum rate of explosion pressure rise ( $\mathrm{kN} / \mathrm{m}^{2} \mathrm{~s}$ )
$V=$ the initial volume occupied by the explosion mixture ( $\mathrm{m}^{3}$ )
$P=$ set operating pressure of the relief vent $\left(\mathrm{kN} / \mathrm{m}^{2}\right)$
$A=$ required relief area ( $\mathrm{m}^{2}$ )
$M=$ the equivalent molecular weight of the gas or vapour
$T=$ the temperature of the vented vapour (K)
$t=$ the time in seconds to attain the maximum pressure at a given rate of pressure rise.
Eqn. 30 was derived by Decker using the ASME flow formulae for mixed vapours [18] (i.e. eqn. 31 and eqn. 29).
$W=13.5 a P(M / T)^{\frac{1}{2}}$
where $W$ is the theoretical flow ( $\mathrm{kg} / \mathrm{s}$ ) and ' $a$ ' is the effective orifice or discharge area ( $\mathrm{m}^{2}$ ) while $R$ or $t$ are derived using the 'cube law' relationship:
$\frac{R}{R_{1}}=\frac{t_{1}}{t}=\left(\frac{V_{1}}{V}\right)^{\frac{1}{3}}$
where $t_{1}, V_{1}$ and $R_{1}$ are the test vessel values and $t, V$ and $R$ are values for the vessel to be protected ( $V$ being limited to the maximum spherical volume possible with the vessel to be protected).

Alternatively $t$ can be determined from a diagram of the average rate of pressure rise $\mathrm{d} P / \mathrm{d} t$ against time, constructed by the method of Latamme [19] using the equation:
$\log P+\log V-\log C=0$
where $C$ is a constant.
One obvious disadvantage of such an approach is the need to measure experimentally $R_{1}, V_{1}$ and $t_{1}$ for every gas or vapour mixture to be employed. Another problem is that the scale and intensity of the turbulence produced in the test vessel must be similar to that in the vessel it is desired to protect, otherwise the 'cube law' will not be applicable as the radial flame speeds will be different. There is also some difficulty in evaluating $T$ and it will normally
have to be estimated by some rule of thumb method. Decker in fact recommends $T$ to be taken as 278 K less than the maximum flame temperature recorded in the combustion literature, or alternatively put equal to 2000 K if that is not known.

Unfortunately there appears to be no experimental evidence as to how well the scaling approach used applies to large volume. In addition since the equation demand that the maximum pressure produced by the explosion is not significantly greater than the operating pressure of the relief, it may well be necessary to operate with unrealistically large vents.

Another approach has recently been developed by Runes [20]. He made the assumption that the maximum rate of pressure rise is produced when the flame reaches the side of the vessel or, in the case of a rectangular structure, when the flame touches the side of minimum dimensions (assuming central ignition).

Given that assumption the eqn. 34 was derived:
$Q_{\mathrm{m}}=A_{\mathrm{m}} V_{\mathrm{f}}\left[\left(M_{\mathrm{f}} / M_{\mathrm{i}}\right)\left(T_{\mathrm{f}} / T_{\mathrm{i}}\right)-1\right]$
where $Q_{m}=$ the maximum rate of increase in the volume of the gas
$A_{m}=$ the maximum area the flame can achieve
$V_{f}=$ the flame velocity
$M_{\mathrm{i}}=$ the initial number of moles of gas
$M_{f}=$ the final number of moles of gas
$T_{\mathrm{i}}=$ the initial gas temperature
$T_{\mathrm{f}}{ }^{0}=$ the adiabatic flame temperature
$A_{\mathrm{m}}$ is $\pi D^{2}$ for a sphere of diameter $D$ and is given as $L_{1} L_{2}$ for a rectangular structure whose smaller and second smallest dimensions are $L_{1}$ and $L_{2}$ respectively, while the term $\left(M_{f} / M_{i}\right)\left(T_{f} / T_{i}\right)$ is approximately equal to the ratio of the final and initial pressure for a completely confined explosion.

The vent is regarded as an orifice discharging unburnt gas and the rate of discharge $Q_{\mathrm{s}}$ is given by:

$$
\begin{equation*}
Q_{\mathrm{s}}=Y C A_{\mathrm{v}}\left(\frac{2 g \Delta P^{\prime}}{D}\right)^{\frac{1}{2}}=Y C A_{\mathrm{v}}\left(\frac{2 \Delta P}{D}\right)^{\frac{1}{2}} \tag{35}
\end{equation*}
$$

where $Y=$ expansion factor
$C=$ the flow or discharge coefficient
$g=$ the acceleration due to gravity
$\Delta P^{\prime}=$ the differential pressure in $\mathrm{kg} / \mathrm{m}^{2}$
$\Delta P=$ the differential pressure in $\mathrm{kN} / \mathrm{m}^{2}$
$D=$ the density of the discharged gas in $\mathrm{kg} / \mathrm{m}^{2}$
$A_{\mathrm{v}}=$ the vent area in $\mathrm{m}^{2}$
$N . B$. this equation is incorrectly presented in Runes original paper since the term $2 \mathrm{~g} / \mathrm{D}$ is left outside of the square root term.

Runes has assumed that for an explosion occurring at ambient temperature and pressure and discharging at high velocities $Y=1$ and $C=0.6$. The density
$D$ has been assumed to have a value of $1.362 \mathrm{~kg} / \mathrm{m}^{3}$ and on that basis eqn. 35 may be reduced to:
$Q_{\mathrm{s}}=0.73 A_{\mathrm{v}} \sqrt{\Delta P}$
The maximum pressure will be attained when $Q_{\mathrm{m}}$ is equal to $Q_{\mathrm{s}}$ in which case:
$A_{\mathrm{V}}=\frac{\pi}{0.73} \frac{L_{1} L_{2}}{\sqrt{\Delta P}} \quad V_{\mathrm{f}}\left[\left(\frac{M_{\mathrm{f}}}{M_{\mathrm{i}}}\right)\left(\frac{T_{\mathrm{f}}}{T_{\mathrm{i}}}\right)-1\right]$
Runes has compared the predicted results for his equation for specific vent areas with the experimental results obtained at the Bromma tests [21] for a $203 \mathrm{~m}^{3}$ chamber. The results are given in Table 3.

As can be seen from Table 3, eqn. 37 overestimates pressures by a factor of up to 2.7. This must be regarded as a serious limitation of the equation as it requires the use of larger vents than are necessary in the structure to be protected.

TABLE 3

| Test <br> no. | Vent <br> area <br> $\left(\mathrm{m}^{2}\right)$ | Measured <br> pressure <br> $\left(\mathrm{kN} / \mathrm{m}^{2}\right)$ | Calculated <br> pressure <br> $\left(\mathrm{kN} / \mathrm{m}^{2}\right)$ |
| :--- | :--- | :--- | :--- |
| 18 a | 21.6 | 3.5 | 9.71 |
| $18 \mathrm{~b}^{*}$ | 21.6 | 5.87 | 9.71 |
| 19 | 17.3 | 6.22 | 15.1 |

*In test 18b a fan was used to promote turbulent conditions.

From eqns. 34 and 35 assuming that $\Delta P$ equals $P_{\text {max }}$ minus the ambient pressure $P$, then we may write:
$P_{\max }=P+\frac{5 D}{Y^{2} C^{2}}\left[\left(\frac{M_{\mathrm{f}}}{M_{\mathrm{i}}}\right)\left(\frac{T_{\mathrm{f}}}{T_{\mathrm{i}}}\right)-1\right]^{2} \quad V_{\mathrm{f}}^{2} K^{2}$
or $P_{\max }=A+B V_{\mathrm{f}}^{2} K^{2}$
where $A=P$, the ambient pressure

$$
\left.B=\frac{5 D}{Y^{2} C^{2}}\left[\left(\frac{M_{\mathrm{f}}}{M_{\mathrm{i}}}\right)\left(\frac{T_{\mathrm{f}}}{T_{\mathrm{i}}}\right)\right]-1\right]^{2}
$$

and is a constant for any particular system.
Thus Runes equation has some similarity to eqns. 26 and 28. The major unknown in eqns. 38 and 39 is $V_{f}^{2}$. Since there is no simple way of determining the scale and intensity of the turbulence produced in an accidental explosion, and the turbulent burning velocity may be up to an order of magnitude greater than the laminar burning velocity which is commonly used,
one might well expect eqns. 37-39 to underestimate the pressure achieved. In fact the reverse appears to be true. Only when conditions are turbulent does the experimental pressure achieved approach those predicted by eqn. 37.

There are two obvious reasons for this discrepancy. The first occurs because the basic assumption that the maximum pressure is achieved when the flame touches vessel walls is not necessarily true. More specifically this assumption is only likely to be true when the second pressure peak, $P_{2}$ is greater than the first, $P_{1}$. The second occurs because the approach ignores the effect of the loss of reactant through the orifice and is therefore likely to overestimate the pressure on this basis as well.

In addition there are other problems with the approach, such as the difficulty of correctly estimating the value of the discharge coefficient $C$ and the fact that these equations have no functional dependence on the operational pressure of the vent. Given these facts it is possibly better to regard Runes equation as predicting an upper limit of $P_{\max }$ rather than the actual value likely to be achieved.

All approaches considered so far either rely on completely empirical relationships or they have to estimate or ignore many parameters. A more rigorous theoretical approach is therefore desirable. An early attempt at producing a rigorous theoretical treatment of the problem of explosion venting was due to Munday [23]. His approach assumes that the gas or vapour burns adiabatically and that the pressure in the vessel is above the critical value for sonic discharge. Given these assumptions a number of different equations were set up and solved by finite difference methods for two geometric shapes, a duct and a sphere. An exact solution of these equations was possible when the ratios of the specific heats for the unburnt and burnt gases were equal and an approximate solution was obtained when they were not.

The equations so produced were compared with measurements made for pentane/air [24] explosions in a duct vented at one end, and also for propane/ air and hydrogen/air explosions [16] in a nearly spherical vessel. The agreement between the theoretical and experimental results were shown to be reasonably good. However, this approach cannot be applied to most practical situations because most industrial and domestic structures must be considered intrinsically weak, and at the critical pressure for sonic discharges most plant and structures would normally be demolished. Thus under atmospheric conditions the criteria for sonic venting, namely that $P_{0}$ is greater or equal to $1.86 P$ would require that the vent operated at $27 \mathrm{lb} / \mathrm{in}^{2}$ or $191 \mathrm{kN} / \mathrm{m}^{2}$. At these pressures Clancey [26] states that one may expect the total destruction of buildings and that heavy machine tools ( $3200 \mathrm{~kg} \mathrm{)} \mathrm{will} \mathrm{be} \mathrm{moved} \mathrm{and} \mathrm{badly}$ damaged. Its use therefore is likely to be restricted to certain 'intrinsically' strong industrial equipment.

Later Munday extended these equations to cover the subsonic venting case by neglecting certain terms in his original equations. This allowed them to be simplified and enabled the vent area for a specific criterion of safety to be determined. This criterion requires that once explosion relief has been
effected, the pressure in the vessel never rises significantly above that level.
The equations for the sonic venting condition which is assumed to occur when the operating pressure of the vent $P_{0}$ is greater or equal to $1.86 \bar{P}, \bar{P}$ being the pressure of the surroundings, into which the explosion vents (usually ambient) are:
$a_{\mathrm{c}}=H b F P_{0}^{\prime}(-\phi)$
$a_{\mathrm{c}}=\frac{A_{\mathrm{c}}}{A_{\mathrm{v}}}$
$P_{0}^{\prime}=\frac{P_{0}}{P_{\mathrm{i}}}$
For the subsonic venting case (which is assumed to occur when $P_{0}$ is less than $1.86 \bar{P}$ ) these equations are:
$a_{c}=H^{\prime} b F P_{c}^{\prime} P_{0}^{\prime}(-\Phi)$
$P_{\mathrm{c}}^{\prime}=\left[\left(\frac{\bar{P}}{P}\right)^{2 / \gamma}-\left(\frac{\bar{P}}{P_{0}}\right)^{\gamma+1 / \gamma}\right]^{1 / 2}$
where: $A_{c} \quad=$ effective area of the vent (i.e. the vent area corrected for real gas flow by the use of a discharge coefficient)
$A_{\mathbf{v}} \quad=$ the area available for positioning the vent (either the total surface area for an approximate spherical vessel or the cross sectional area of an elongated vent or duct)
$\bar{P} \quad=$ the pressure of the surrounding atmosphere into which the explosion is vented
$P_{0} \quad=$ the operating pressure of the relief
$P_{\mathrm{i}} \quad=$ the initial pressure in the vessel
$\gamma \quad=$ ratio of specific heats
$H, H^{\prime}$ and $\phi=$ rates of pressure rise terms (with $H$ and $H^{\prime}$ showing a dependence on a turbulence factor $f$ and the fundamental flame speed under initial conditions)
$F \quad=$ configuration/ignition factor $(F=1,2$ or 4 for central ignition in a nearly spherical vessel, ignition at one end and central ignition in an elongated vessel respectively).
Also:
$b=\frac{P_{\mathrm{e}}-P_{\mathrm{s}}}{P_{\mathrm{s}}}$
where: $P_{\mathrm{e}}=$ the maximum pressure attained in the closed vessel explosion
$P_{\mathrm{s}}=$ the pressure at start
There is unfortunately an important limitation of Munday's equation for the subsonic venting case. This limitation is imposed by the criterion that the
explosion pressure does not significantly rise above the vent operating pressure. In practice this will mean the vent size must be adjusted in order to ensure the first pressure peak $P_{1}$ is always the greatest, as well as preventing the magnitude of this peak from becoming significantly larger than that of the operating pressure.

To achieve this it may well be necessary either to use vents which have a higher operating pressure than is necessary and therefore reduce the degree of safety, or alternatively to use vents that are larger than otherwise would be necessary. This restriction may well mean that Munday's approach is of limited value in many circumstances where it is not technically feasible to use 'very' large vents.

One of the most recent approaches is due to Yao et al. [27,28,29] who have produced a set of differential equations to describe the behaviour of vented explosions based on the law of conservation of mass and the assumption of adiabatic compression. The model also takes into account the dependence of burning velocity on pressure and temperature and the effects of high initial pressure and sonic discharge on the pressure development.

These equations are*:
(1) Dimensionless rate of change in pressure in the enclosure:
$\begin{aligned} & \frac{\mathrm{d} \zeta}{\mathrm{d} \tau}=\left\{3 \chi \gamma(\nu-1) \zeta^{(3 \gamma-2) / 3} \xi_{\xi^{2 / 3}}\left(\frac{T_{\mathrm{u}(0)}}{T_{\mathrm{u}(r)}}\right)^{2}\left(\frac{P_{(r)}}{P_{(0)}}\right)^{\beta} \zeta^{(2 \gamma-2-\gamma \beta) / \gamma}\right. \\ &\left.-\alpha\left(\nu \psi_{\mathrm{b}}+\nu^{1 / 2} \psi_{\mathrm{u}}\right) \gamma \zeta^{(\gamma-1) / \gamma} E\right\}\end{aligned}$
(2) Dimensionless rate of change in burned gas in the enclosure:
$\frac{\mathrm{d} \xi}{\mathrm{d} \tau}=\left\{3 \chi \zeta^{1 / 3 \gamma} \xi^{2 / 3}\left(\frac{T_{\mathrm{u}(0)}}{T_{\mathrm{u}(r)}}\right)^{2}\left(\frac{P_{(r)}}{P_{(0)}}\right)^{\beta} \zeta^{(2 \gamma-2-\gamma \beta) / \gamma-\alpha \psi_{\mathrm{b}} E}\right\}$
(3) Dimensionless rate of change in unburned gas in the enclosure:
$\frac{\mathrm{d} \lambda}{\mathrm{d} \tau}=-\left\{3 \chi \zeta^{1 / 3 \gamma} \xi^{2 / 3}\left(\frac{T_{\mathrm{u}(0)}}{T_{\mathrm{u}(r)}}\right)^{2}\left(\frac{P_{(r)}}{P_{(0)}}\right)^{\beta} \zeta^{(2 \gamma-2-\gamma \beta / \gamma)}+\alpha \psi_{u^{\prime}} \nu^{1 / 2} E\right\}$
For subsonic flow:
$E=\sqrt{\frac{\gamma}{\gamma-1}\left(\frac{P_{(\mathrm{a})}}{P_{(0)}}\right)^{(\gamma+1) / \gamma}\left[\left(\frac{P_{(0)}}{P_{(\mathrm{a})}} \zeta\right)^{(\gamma-1) / \gamma}-1\right]}$
For the sonic flow case, when: $\zeta>\frac{P_{(\mathrm{a})}}{P_{(0)}}\left(\frac{\gamma+1}{2}\right)^{\gamma /(\gamma-1)}$

[^1]$E=\frac{1}{\sqrt{2}}\left[\left(\frac{2}{\gamma-1}\right)^{(\gamma+1) /(\gamma-1)} \gamma \zeta^{(\gamma+1) / \gamma}\right]^{1 / 2}$
Dimensionless pressure:
$\zeta=\frac{P_{(t)}}{P_{(0)}}$
Dimensionless time:
$\tau=\tau \nu^{2 / 3} s / a$
Dimensionless burned gas remaining:
$\xi=M_{r b}(t) / M_{0}$
Dimensionless unburned gas remaining:
$\lambda=M_{r u(t)} / M_{0}$
Dimensionless density ratio:
$\nu=M_{u} T_{\mathrm{b}(0)} / M_{\mathrm{b}} T_{\mathrm{u}(0)}$
or:
$\nu=\rho_{\mathbf{u}(0)} / \rho_{\mathrm{b}(0)}$
Explosion venting parameter for a spherical vessel:
$\alpha=\frac{\sqrt{2} C A_{v} a}{S V}\left(\frac{P_{(0)}}{\rho_{\mathrm{u}(0)}}\right)^{1 / 2}\left(\frac{1}{V}\right)^{7 / 6}$
where: $V=$ the volume of the enclosure
$a \quad=$ the equivalent radius of the enclosure
$A_{v}=$ the vent area
$M_{r \mathrm{~b}}=$ the burned gas remaining in the enclosure
$M_{r u}=$ the unburned gas remaining in the enclosure
$M_{0}=$ the initial gas mass in the enclosure
$P_{(0)}=$ the initial pressure
$P_{(a)}=$ the ambient pressure outside the enclosure
$P_{(r)}=$ the reference pressure (normally atmospheric pressure) the burning velocity $S$ is measured.
$P(t)=$ the pressure inside the enclosure at any time
$P_{(m)}=$ the maximum pressure in a closed vessel
$\rho_{\mathrm{u}(0)}=$ the density of unburned gas at initial pressure
$\rho_{\mathrm{b}(0)}=$ the density of the burned gas at initial pressure
$x$ = the turbulence correction factor
\[

\left.$$
\begin{array}{rl}
\psi_{\mathrm{b}}, \psi_{\mathrm{u}}= & \text { the fraction of the total opening area from which the burned } \\
& \text { and unburned gases are flowing out respectively. } \\
T_{\mathrm{u}(t)}= & \text { the absolute temperature of the unburned mixture at any } \\
& \text { time after ignition }
\end{array}
$$\right] $$
\begin{aligned}
T_{\mathrm{u}(r)}= & \text { the reference temperature at which } S \text { is determined } \\
\beta & =\text { an exponent indicating the dependence of the burning velocity } \\
& S \text { on pressure }
\end{aligned}
$$
\]

By setting the venting parameter $\alpha$ equal to zero Yao was able to show that one could satisfactorily describe explosion development in closed vessels, which indicates the versatility of this approach. More importantly, with the correct choice of turbulence factor $\chi$, the equations were able to duplicate the pressure time curves for a 5 per cent propane/air explosion in a $0.76 \mathrm{~m}^{3}$ cubical enclosure, two 0.91 m diameter cylindrical chambers of 0.91 and 8.2 m length respectively, including the double pressure peak phenomena. This fact probably makes Yao's approach the most promising of those now available, although it has been tested on the small scale only.

For computational purposes Yao et al. have chosen to set:

$$
\begin{align*}
& \psi_{\mathbf{u}}=\lambda /(\xi+\lambda)  \tag{58}\\
& \psi_{\mathrm{b}}=\xi /(\xi+\lambda) \tag{59}
\end{align*}
$$

This choice must be regarded as somewhat arbitrary, ignoring as it does the geometric and other factors which will govern the degree of mixing of unburnt and burnt gases and their relative concentrations at the vent. However this will hold good at the limits (i.e. at the beginning and end of the explosion) and represents a simple solution to what otherwise might prove an intractable problem.

There exists, however, one major weakness in the method which is the need to choose the correct value of the turbulence factor $\chi$, since that choice is at the moment arbitrary. It is also unlikely that the scale and intensity of turbulence remains constant through an explosion, although to a first approximation this may be so. In addition, Yao himself reports that the second peak is more sensitive to the effects of turbulence [28,29] in which case greater care will be necessary in evaluating $\chi$ when the second pressure peak is likely to be the largest.

Another feature of interest in Yao's work is the choice of venting parameter. Yao has used a venting parameter $\alpha$ which is related to the venting area parameter $G$ in the following manner:
$\alpha=$ constant $\times\left[\frac{a A_{\mathbf{v}}}{V}\right]=$ constant $\times[G]$
For a spherical enclosure:
$G=\frac{a A_{\mathrm{v}}}{V}=3 \frac{A_{\mathrm{v}}}{A_{\mathrm{T}}}$
where $A_{T}$ is the total surface area. Hence $\alpha$ is related to $K$, i.e. for a spherical enclosure:
$G=3 / K$
For cubical enclosure:
$G=\frac{a A_{\mathbf{v}}}{V} \approx \frac{4 A_{\mathbf{v}}}{A_{\mathbf{T}}}=\frac{2}{3 A / A_{\mathbf{v}}}=\frac{2}{3 K}$
where $A=$ area of a side of the enclosure.
Thus $\alpha$ is a dimensionless parameter of the system and may be related to some constant divided by $K$.

A similar approach to that of Yao et al. is due to Pasman et al. [30]. These workers have also produced a set of differential equations to describe the behaviour of vented explosions based on the law of conservation of mass and the assumption of adiabatic compression. Unlike Yao they have considered the case of explosions in cylindrical vessels vented at one end and have assumed that only unburnt gas is vented. The flame ball upon touching the side of the vessel, is considered to develop as two constant hemispherical surfaces travelling in opposite directions, central ignition being assumed. The actual equations will not be presented here as they are not given in the final form in which they are solved.

To verify their approach experiments have been carried out on stoichiometric methane/air and hydrogen/air mixtures in a $1 \mathrm{~m}^{3}$ cylindrical vessel as developed by Bartknecht [31]. Reasonable agreement between experimental and theoretical work with the appropriate choice of turbulence factor is reported.

This work is open to the same criticisms as that of Yao et al. namely the experiments have been carried out on the small scale only and also the approach requires the use of a turbulence factor which must be determined empirically.

Recently Nettleton [32] has attempted to produce an analysis of vented explosions using the methods of characteristics [33] to describe the behaviour of pressure waves produced by bursting diaphragms in differently shaped vessels. The suggestion is also made that the reflection of the expansion fan may partly explain the existence of the double pressure peak phenomena so far observed with vented explosions.

The work carried out has employed two tanks (one 47 mm square cross section and 300 m long and the other 47 by 109 mm in cross section and 500 mm long) as the vented vessels. Both of these were connected to sections of tube (of cross-section $47 \times 22 \mathrm{~mm}$ ) either by means of an abrupt or gradual transmission section which contained a number of flanges to allow a cellophane diaphragm to be positioned at different distances down the tube. The tank and the associated portions of the tube were slowly pressurised until the diaphragm burst, whereupon the pressure history in the tank was followed by means of pressure transducers situated at various points. In order to ob-
tain reproducible results, Nettleton reports that it was necessary to evacuate the low pressure side of the diaphragm to a few hundred $\mathrm{N} / \mathrm{m}^{2}$, when this procedure was followed a double peak phenomena occurred within approximately 5 ms of the rupture of the diaphragm.

In considering this work a number of points may be raised. The first is that it is difficult to see what formal relationship Nettleton's system has with those employed by other workers such as Rasbash [12] and Yao [29]. The normal deflagration explosion event considered in venting problems is associated with a continuously expanding flame front whose speed is such that the transmission of pressure can be considered effectively instantaneous. Nettleton on the other hand considers the decay of a fixed pressure in a closed system due to an expansion wave, the double pressure peak being due to the reflection of the wave. Thus his overall system seems more reminiscent of a shock wave system than a normal deflagration explosion.

Nettleton refers to the pressure records obtained by Burgoyne and Wilson [15] and Harris and Briscoe [6] from vented explosions which show an initial small peak followed by a second peak of much greater amplitude. On examination of Burgoyne and Wilson's work it would appear that a multiple pressure peak phenomena occurring under highly turbulent conditions with a spring-loaded plate valve is being referred to (i.e. Figure 5 in Burgoyne and Wilson's paper [15]) while in the case of Harris and Briscoe it appears that once again a multiple pressure peak phenomena is being discussed, which also occurs under highly turbulent conditions (i.e. Figure 1(c) of Harris and Briscoe's paper [6]) where the assumption that pressure transmission is effectively instantaneous may well be questionable.

However, Harris and Briscoe have shown another pressure record in which the first peak is greater than the second and the two peaks are separated by 120 ms (i.e. their Figure 1(b)) conditions under which the second peak is unlikely to be due to the reflection of an expansion wave [6]. Generally this type of phenomenon is more typical of vented deflagration explosions, where the two peaks may change their respective sizes and tend to be separated by time intervals of 100 ms or more. Thus whereas Nettleton's explanation for the double pressure peak phenomenon he observes in his own experiments is no doubt valid, and may well be the correct explanation for the pressure spiking so often observed on explosion pressure-time records [5], it would appear likely that the double pressure peak phenomenon frequently observed with vented explosions arises for fundamentally different reasons and is a different phenomenon from the one he describes.

The conditions and scale of Nettleton's work are so different from those normally employed that it will take a considerable time before the full implications of his work on explosion venting problems are likely to be demonstrated. Thus although Nettleton's treatment is clearly valid for the system he considered, it does not seem immediately applicable to practical domestic or industrial venting problems.

Nettleton has also produced a guide to the empirical rules used in venting explosions [34] with particular reference to "volumetric explosions". These are described as explosions in which the burning time of the particles is similar to the ratio of the characteristic length of the vessel to the burning velocity of the cloud. Typical pressure profiles are characterised by an initial slow development followed by an increasingly rapid pressure rise, which will be 10 to 100 times that of the initial slow rate, the pressure finally falling off again to produce an 'S' shaped curve. The maximum pressure ratio associated with such a process will be approximately equal to the ratio of the temperature of the burnt gases to that of the unburnt gases.

Nettleton indicates that time for the expansion wave (i.e. a negative $\mathrm{d} P / \mathrm{d} t$ wave) to travel to the further extremity of the vessel is of great importance, in these explosions. Thus if the time for the vent operating pressure and opening becomes greater than the period of slow pressure rise then the pressure may rise at the furthest extremity of the vessel, at a sufficiently fast rate to damage the vessel before the expansion wave arises. In that event all approaches which assume effectively instantaneous pressure transmission will be invalid and Nettleton recommends instead the use of the method of characteristics to determine the pressure-time characteristics of the explosions as outlined in the earlier paper [32].

As an illustration of this type of danger Nettleton has calculated the minimum time $\tau_{\text {min }}$ for venting to occur for a hypothetical volumetric explosion. The explosion occurs in a 3 m long 1 m diameter cylindrical vessel vented at one end by a vent of $1.5 \mathrm{~m}^{2}$. The maximum permissible pressure is given as $150 \mathrm{kN} / \mathrm{m}^{2}$ and the vent operating pressure is $120 \mathrm{kN} / \mathrm{m}^{2}$, the explosion is considered to produce a slow rate of pressure rise of $200 \mathrm{kN} / \mathrm{m}^{2} / \mathrm{s}$ for the first 100 ms followed by a fast rate of pressure rise of $5000 \mathrm{kN} / \mathrm{m}^{2} / \mathrm{s}$. Assuming $\tau_{\min }$ to be the sum of the times for the vent operating pressure to be achieved, the vent to operate and an expansion wave to travel to the furthest extremity of the vessel, Nettleton is able to show that for likely values of $\tau_{\min }$ the pressure at the furthest portion of the vessel will rise to $160 \mathrm{kN} / \mathrm{m}^{2}$ or more.

An important comment that can be made about this calculation is that the choice of figures for the maximum permissible pressure and the vent operating pressure are appropriate only to intrinsically strong plant. Thus in most domestic and industrial situations the probability is that the vent operating pressure will be achieved well within the period of low pressure rise and the assumption that pressure is transmitted effectively instantaneously will be good.

## Conclusion

Of all the approaches so far examined that of Yao [27-29] seems most promising since it has theoretical justification and is the most versatile approach of those examined. Its major weakness is the need to introduce an
arbitrary turbulence factor as a multiplier to the burning velocity. The production of an adequate mathematical model for confined deflagration explosion will therefore depend on resolving the problem of turbulence. Yao, however, has incorporated most factors for the production of an adequate formula and it would therefore seem worthwhile to develop this approach.

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[^0]:    *Spiking is a series of rapid excursions of pressure about the mean pressure time curve.

[^1]:    *These equations are incorrectly presented in the original papers.

